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A DECISION OF SIMPLE HARMONIC MOTION AND ITS METHOD

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ABSTRACT:

We study of simple harmonic motion and its applications. The simple harmonic motion of the spring block system generally shows a behavior that is strongly influenced by the geometric parameters of the spring. In this work, we study the oscillatory behavior of a spring mass system, considering the effect of changing the average diameter of the spring on the elastic constant k, the angular frequency ω, the damping factor γ and the dynamics of the oscillations. Simple harmonic motion and get expressions for the velocity, acceleration, amplitude, frequency, and position of the particle performing this motion. Its applications are watches, guitars, violins, bungee jumping, rubber bands, trampolines, earthquakes or the problems discussed. Key words: acceleration, amplitude, angular frequency, velocity.

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1. INTRODUCTION:

In mechanics and physics, simple harmonic motion is a type of periodic motion or oscillatory motion in which the restoring force is directly proportional to the displacement and acts in the opposite direction to the displacement. Simple harmonic motion can serve as a mathematical model for a variety of motions, such as the oscillation of a spring. In addition, other phenomena can be approximated by simple harmonic motion, including the motion of a simple pendulum as well as molecular vibration. Simple harmonic motion is the motion of a mass on a spring when subjected to the linear elastic restoring force given by Hooke's law. The motion is sinusoidal in time and shows a single resonant frequency. For simple harmonic motion to be an accurate model of a pendulum, the total force on the body at the end of the pendulum must be proportional to the displacement. This will be a good approximation when the swing angle is small. Simple harmonic motion provides a basis for characterizing more complex motions through Fourier analysis techniques. The motion of a particle moving along a straight line with acceleration always in the direction of a fixed point on the line and whose magnitude is proportional to the distance from the fixed point is called simple harmonic motion. In the diagram, a simple harmonic oscillator is shown, consisting of a weight attached to one end of the spring. The other end of the spring is attached to a solid bracket like a wall. If the system is left at rest in equilibrium, there is no net force acting on the mass. However, if the mass is displaced from its equilibrium position, the spring exerts a restored elastic force that adheres to Hooke's law.

Mathematically, the restoring force F is given by

$$F= -Kx$$

Where F is the restoring elastic force exerted by the spring (in SI units: N), k is the spring constant ($N \cdot m^{-1}$), and x is the displacement from the equilibrium position (m).

2. HEADINGS

The equation of motion of a particle that executes a simple harmonic motion, Geographical representation of simple harmonic motion, Composition of two simple harmonic motions of the same period along the same straight line, Composition of two simple harmonic motions of the same period into two perpendicular directions.

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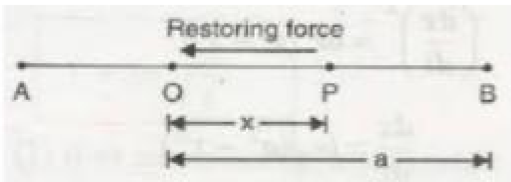
3. INDENTATIONS AND EQUATIONS



The Equation of Motion of a Particle Executing Simple Harmonic Motion

Let O be the fixed point on the straight line AOB at which a particle has simple harmonic motion. Take O as the origin and OA as the X axis. Let P be the position of the particle at time 't' such that OP = x. The magnitude of the acceleration at P = μx where μ is a positive constant. Since this acceleration acts toward O, the acceleration at P in the positive x-axis direction is $-\mu x$. The magnitude of the acceleration at P is proportional to x which is the magnitude of the acceleration is μx where μ is a constant. As the acceleration is directed toward O. (that is, in the direction of decreasing x). Therefore, the equation of motion of P is,

$$d^2x/dt^2=-\mu x \rightarrow (1)$$

Equation 1 is the fundamental differential equation representing a simple harmonic motion. We now proceed to solve it. If V-velocity of the particle at time „t", 1 can be written as

$$Vdv/dx=-\mu x$$

$Vdv=dx.\ -\mu x \rightarrow (2)$

$V^2/2=-\mu x^2/2+c \rightarrow (3)$

Initial value x=a, v=0, Put in equation 3,

$V^2/2=-\mu x^2/2+c$

$0=-\mu a^2/2+c$

$c=-\mu a^2/2$

$V^2/2=-\mu x^2/2+\mu a^2/2$

$V^2=-\mu x^2+\mu a^2$

$V^2=\mu(a^2-x^2)$

$\therefore\ V=\sqrt{\mu(a^2-x^2)} \rightarrow (4)$

Equation 4 gives the velocity v corresponding to any displacement x. Now as „t" increases, x decreases.

So,dx/dt is negative.

$dx/dt=V=-\sqrt{\mu(a^2-x^2)} \rightarrow (5)$

$dx/dt=-\sqrt{\mu(a^2-x^2)}$

$-dx/\sqrt{\mu(a^2-x^2)}=\sqrt{\mu}.dt$ Integrating,

$\cos^{-1} x/a=\sqrt{\mu}.t+A$

Initially when t=0,x=a

$\cos^{-1}=0+A$

Hence

$\cos^{-1} x/a=\sqrt{\mu}.t$

$x/a= \cos\sqrt{\mu}.t$

$x=a \cos\sqrt{\mu}.t$

I.GEOMETRICAL REPRESENTATION OF SIMPLE HARMONIC MOTION

Show that if a particle describes a circle with constant angular velocity, then the foot of the perpendicular on a diameter moves with simple harmonic motion.

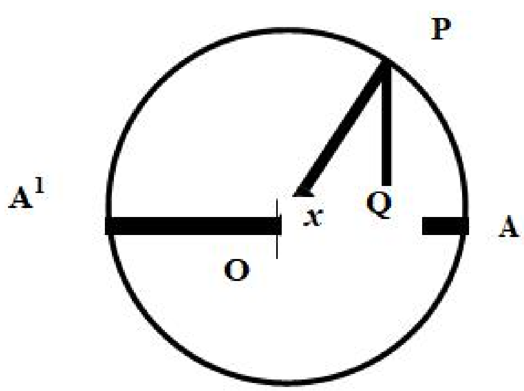


Fig No 1: GEOMETRICAL REPRESENTATION OF SIMPLE HARMONIC MOTION

Let a particle move along the circumference of circle of radius a with uniform angular velocity ω. let AA^1 be a diameter of the circle. Let the position of the particle at time „t" be P. Then t .ω=AOP ∠ Draw PQ ⊥r to AA1 and let OQ=x. Then (1)→t ω cos=x

As P moves on the circle Q moves o the diameter AA^1 that is Q oscillates between A and A^1 along AA^1 . Therefore the motion of Q is simple harmonic motion. From (1)

$dx/dt=-a\ \omega \sin \omega t \rightarrow (2)$

$d^2x/dt^2=-a\ \omega^2 \cos \omega t$

$d^2x/dt^2= \omega^2 x \rightarrow (3)$

Equation (2) gives the velocity of the particle Q and (3) gives the acceleration of Q at time T. Also from (3) we note that the motion of Q is simple harmonic. We know that the amplitude of the simple harmonic motion is a.

The periodic time of $Q=2\pi/\omega$

If a particle describes a circle with constant angular velocity then the foot of the perpendicular from it on any diameter executes simple harmonic motion.

II.Composition of Two Simple Harmonic Motions of the Same Period along the Same Straight Line

Let the two simple harmonic motions of the same period be given by

$x=a \cos(\sqrt{\mu}\ t+\epsilon_1)$

$x=b \cos(\sqrt{\mu}\ t+\epsilon_2)$

The composition of the two simple harmonic motions is

$x=a \cos(\sqrt{\mu}\ t+\epsilon_1)+ b \cos(\sqrt{\mu}\ t+\epsilon_2)$

$\cos(A+B)=\cos A.\cos B - \sin A.\sin B$

$x=a[\cos\sqrt{\mu}t.\cos \in_1 - \sin\sqrt{\mu}t.\sin \in_1]+b[\cos\sqrt{\mu}t.\cos \in_2 - \sin\sqrt{\mu}t.\sin \in_2]$

$x=a\cos\sqrt{\mu}t.\cos \in_1 - a\sin\sqrt{\mu}t.\sin \in_1 + b\cos\sqrt{\mu}t.\cos \in_2 - b\sin\sqrt{\mu}t.\sin \in_2$

$x=[a \cos \in_1 + b \cos \in_2] \cos\sqrt{\mu}t -[a \sin \in_1 +b \sin\in_2]\sin\sqrt{\mu}t$

$a \cos \in_1 \sqrt{\mu}t\ b \cos \in_2 = A \cos \in$ (I)

$a \sin \in_1 \sqrt{\mu}t\ b \sin \in_2 = A \sin \in$ (II)

Then,

$x = A \cos \in\cos \sqrt{\sqrt{\mu}t}\ A\sin\in\sin\sqrt{\mu}t$

$x=A(\cos \in\cos\sqrt{\mu}t -\sin\in\sin\sqrt{\mu}t)$

$x=A\cos\sqrt{\mu}t +\in)$

This equation shows that the composition of two simple harmonic motions is also a simple harmonic motion with the same period.

A is the amplitude and $\in$is the epoch

Dividing(2) by (1),

$A\sin\in/ A\cos\in= a\sin\in_1 b\sin\in_2/ A\cos\in_1+ b\cos\in_2$

$\tan = a\sin\in_1+b\sin\in_2/ A\cos\in_1+ b\cos\in_2$

Squaring and adding (I) and (II),

$A^2 \cos^2 \in+ A^2 \sin^2 \in=(a \cos \in_1 +b \cos \in_2)^2 + (a \sin \in_1 +b \sin \in_2)^2$

$A^2(\cos^2\in + \sin^2 \in)= a^2\cos^2 \in_1+b^2 \cos^2 \in_2+2a\cos\in_1.b\cos\in_2+a^2\sin^2\in_1+b^2\sin^2\in_2+2a \sin \in_1\sin$ $A^2(\cos^2\in + \sin^2 \in)= a^2\cos^2 \in_1+b^2 \cos^2 \in_2+2a \cos\in_1.b \cos \in_2+a^2 \sin^2 \in_1+ b^2 \sin^2 \in_2+2a \sin \in_1\sin \in_2$

$A^2=a^2(\cos^2\in_1 +\sin^2 \in_1) +b^2(\cos^2 \in_2+\sin^2 \in_2) +2ab[\cos \in_1 . \cos \in_2 + \sin \in_1 . \sin \in_2]$

$A^2=a^2+b^2+2ab\cos(\in_1-\in_2)$

$A= \sqrt{a2+b2+2ab\cos(\in_1-\in_2)}\rightarrow(4)$

(3) gives $\in$ and (4) gives the amplitude A

CONCLUSION

In this case, study simple harmonic motion and its applications. The problems of different applications are solved analytically with the exact equation of simple harmonic motion. We can calculate the periodic time value of oscillating an object from the origin using these methods.

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